International Journal of Theoretical Physics, Vol. 7, No. 6 (1973), pp. 391–414

The Physics of a Metric Space with a Time Variable

ROBERT L. KIRKWOOD

1355 *Berea Place, Pacific Palisades, California* 90272, *U.S.A.*

Received: 4 *April* 1972

Abstract

It is shown that the introduction of a time variable in a curved metric four-space of the Einstein type leads to an interpretation of gravity as an ether flow in a Riemannian three-space. It is assumed that the only motion that enters into physical laws is either motion relative to the ether or the relative motion of nearby points that are fixed in the ether, and this assumption is formulated analytically. A previous formulation of Newtonian fields in a metric four-space indicates that the three-space can be assumed to be Euclidean and provides field equations to determine the motion of the ether. It is also suggested that the velocity of light relative to the ether has the constant value c in many important physical fields. Finally, the observer's coordinates of the special theory of relativity are defined in the presence of a gravitational field.

1. *Introduction*

Most modern gravitational theories are written in terms of a curved metric four-space in which the paths of particles and light rays are assumed to be the geodesics of the metric tensor. However, the physicist who uses these theories has been conditioned from birth by a classical space-time, in which it appears that there is a universal Newtonian time and that three-dimensional space is Euclidean. As a result, much of the physical intuition that he develops in his everyday life is not readily applicable to modern gravitational theory. Since physical intuition is one of his most valuable guides to the advancement of physics, it is very desirable to describe a curved metric space in the everyday terms of classical physics, and it is the object of this paper to show how this can be done.

Consider a four-space in which an arbitrary set of space-time coordinates x_α has been introduced. In principle this could be done by building a three-dimensional lattice which is moving or deforming in any way and by hanging a clock at each lattice point. Each lattice point can be described by three spatial coordinates, and the clocks can be synchronized in any way, provided only that the resulting coordinates have unique values for any space-time event that may be considered. A physical clock which moves through a space-time interval dx_α will be assumed to measure a time $d\tau$ that can be expressed in the form

$$-c^2\,d\tau^2 = g_{\alpha\beta}\,dx_\alpha\,dx_\beta \tag{1.1}$$

(Here, and throughout this paper, Greek indices will run from 1 to 4, Roman indices from 1 to 3, and repeated indices will be summed throughout their range.) The coefficients $g_{\alpha\beta}$ are assumed to be of the type considered by Einstein, which can be diagonalized to have the diagonal values 1, 1, 1, and -1. The factor $-c^2$, where c is the velocity of light, is inserted in equation (1.1) so that a positive value of $g_{\alpha\beta}dx_\alpha dx_\beta$ will represent the square of a distance, while a negative value will have a magnitude which represents the square of c times a local time interval. The time-like coordinate will be denoted by x_4 and will be assumed to satisfy the inequality

$$g^{\alpha\beta}\frac{\partial x_4}{\partial x_\alpha}\frac{\partial x_4}{\partial x_\beta} \equiv g^{44} < 0 \tag{1.2}$$

where $g^{\alpha\beta}$ is the reciprocal of $g_{\alpha\beta}$. This inequality implies that there is a positive real function ζ such that

$$g^{\alpha\beta}\frac{\partial x_4}{\partial x_\alpha}\frac{\partial x_4}{\partial x_\beta} \equiv g^{44} = -\frac{1}{\zeta^2} \tag{1.3}$$

In three-dimensional notation equation (1.1) can be rewritten

$$-c^2\,d\tau^2 = g_{ij}\,dx_i\,dx_j + 2g_{i4}\,dx_i\,dx_4 + g_{44}\,dx_4{}^2 \tag{1.4}$$

where the sums are now carried from 1 to 3. The meaning of this relation can be seen in classical terms if a three-dimensional tensor h^{ij} is defined in these coordinates to be the reciprocal of the three-dimensional tensor g_{ij}, so that $h^{ij}g_{jk} = \delta_k{}^i$, and if the quantities g_{i4} are written in terms of the components of a three-dimensional vector field v^i, defined by

$$v^i \equiv -h^{ij}\,g_{j4} \tag{1.5}$$

Then equation (1.4) becomes

$$-c^2\,d\tau^2 = g_{ij}(dx_i - v^i\,dx_4)(dx_j - v^j\,dx_4) + (g_{44} - g_{ij}v^i v^j)\,dx_4{}^2 \tag{1.6}$$

By direct calculation, noting that g_{i4} is given in terms of v^i by equation (1.5), it is found that the determinant of the four-dimensional metric tensor $g_{\alpha\beta}$ is

$$g = h(g_{44} - g_{ij}\,v^i\,v^j)$$

where h is the determinant of the three-dimensional tensor g_{ij}. From this it is clear that the quantity $g_{44} - g_{ij}v^i v^j$ in equation (1.6) is equal to g/h. Since h is the cofactor of g_{44} in the determinant g, h/g is g^{44}, which by equation (1.3) is $-1/\zeta^2$, so that

$$g_{44} - g_{ij}\,v^i\,v^j = \frac{g}{h} = -\zeta^2 \tag{1.7}$$

and equation (1.6) becomes

$$-c^2\,d\tau^2 = g_{ij}(dx_i - v^i\,dx_4)(dx_j - v^j\,dx_4) - \zeta^2\,dx_4^{\,2} \tag{1.8}$$

Along the path of a light ray $d\tau$ is assumed to vanish, and equation (1.8) becomes

$$g_{ij}\left(\frac{dx_i}{dx_4} - v^i\right)\left(\frac{dx_j}{dx_4} - v^j\right) = \zeta^2$$

Since g_{ij} is the three-dimensional metric tensor on a surface defined by a given value of x_4, and dx_i/dx_4 is the velocity of the light ray in the coordinates x_α, this equation says that the speed of light relative to a point which moves with velocity v^i is the same in all directions and has the magnitude ζ. Thus *the velocity field v^i plays the role of the classical ether velocity* in that the velocity of light relative to the ether will then have the same value ζ in all directions.

Along a path on which $d\tau$ does *not* vanish, equation (1.8) shows that the three-dimensional displacement dx_i enters into the expression for $d\tau$ only in the form of $dx_i - v^i\,dx_4$, that is, *the only displacement of a moving clock which affects its measured time is its displacement relative to the ether.* Since the paths of particles and light rays are assumed to be the geodesics of this measured time, *the only motion of particles or light rays that enters into the laws of mechanics or of ray optics is motion relative to the ether.* As a result, the ether plays the role of the primary inertial system of classical physics, differing from it only in that it is not generally possible to introduce Cartesian coordinates in which the ether is everywhere at rest.

This description of a metric space in terms of an ether flow is strongly dependent on the particular variable x_4 that is chosen as a measure of time. If it is assumed, as in Newtonian theory, that there is a single universal time variable, then the motion of the ether is uniquely determined. However, in the special theory of relativity it is assumed that there are many different time variables, each of which is an equally valid measure of time, and in this situation there is a different ether velocity field for each such time variable. The ether velocity fields associated with two different time variables can be related by finding the ether velocity that would exist in some new set of coordinates x_α' and determining the motion of a point that is fixed in the ether in these new coordinates relative to the ether in the original coordinates x_α. To do this, it is first observed that the reciprocal of $g_{\alpha\beta}$ is $g^{\alpha\beta}$, whose components are given by

$$\begin{aligned} g^{ij} &= h^{ij} - \frac{v^i v^j}{\zeta^2} \\ g^{i4} &= -\frac{v^i}{\zeta^2} \\ g^{44} &= -\frac{1}{\zeta^2} \end{aligned} \tag{1.9}$$

To prove that this is the correct form of $g^{\alpha\beta}$, it is only necessary to note that $g_{i4} = -g_{ij}v^j$ by equation (1.5) and $g_{44} = -\zeta^2 + g_{ij}v^i v^j$ by equation (1.7), and it can then be shown by direct calculation that $g^{\alpha\beta}g_{\beta\gamma} = \delta_\gamma{}^\alpha$. From equations (1.9) it is seen that the ether velocity v^i in the coordinates x_α can be written

$$v^i = \frac{g^{i4}}{g^{44}} \tag{1.10}$$

Since the coordinates x_α were arbitrary except that x_4 is time-like, the ether velocity $v^{i\prime}$ in the new coordinates $x_\alpha{}'$ can be written similarly in terms of the $g^{\alpha\beta\prime}$, and is

$$v^{i\prime} = \frac{g^{i4\prime}}{g^{44\prime}} \tag{1.11}$$

Points that are fixed in the ether in the new coordinates are those whose coordinate differentials satisfy the relation

$$dx_i{}' = v^{i\prime}\, dx_4{}' \tag{1.12}$$

or, from equation (1.11),

$$dx_\alpha{}' = g^{\alpha 4\prime}\frac{dx_4{}'}{g^{44\prime}} \tag{1.13}$$

The index i is replaced here by α and allowed to run from 1 to 4 because the equation is trivially satisfied for $\alpha = 4$. Multiplying equation (1.13) by $\partial x_\beta / \partial x_\alpha{}'$ and using the usual transformation relations for the quantities $g^{\alpha\beta\prime}$ and $dx_\alpha{}'$ gives

$$dx_\beta = g^{\beta\gamma}\frac{\partial x_4{}'}{\partial x_\gamma}\frac{dx_4{}'}{g^{44\prime}} \tag{1.14}$$

If a point which is fixed in the ether of $x_\alpha{}'$ moves with velocity W^i relative to the ether of x_α, then, by definition,

$$W^i \equiv \frac{dx_i}{dx_4} - v^i \tag{1.15}$$

where dx_i and dx_4 are given by equation (1.14), that is,

$$W^i = \frac{g^{i\beta}\,\partial x_4{}'/\partial x_\beta}{g^{4\gamma}\,\partial x_4{}'/\partial x_\gamma} - v^i \tag{1.16}$$

If the quantities $g^{i\beta}$ and $g^{4\gamma}$ are evaluated from equations (1.9) and if the total time derivative at a point moving with the ether is denoted by d_e/dx_4, so that

$$\frac{d_e}{dx_4} \equiv \frac{\partial}{\partial x_4} + v^i\frac{\partial}{\partial x_i} \tag{1.17}$$

equation (1.16) becomes

$$W^i = -\frac{\zeta^2}{d_e x_4{}'/dx_4}\, h^{ij}\frac{\partial x_4{}'}{\partial x_j} \tag{1.18}$$

The meaning of equation (1.18) is very clear when it is applied to the familiar flat-space metric of the special theory, for which the coordinates x_α can be chosen so that

$$\begin{aligned} g_{ij} &= \delta_{ij} \\ g_{i4} &= 0 \\ g_{44} &= -c^2 \end{aligned} \tag{1.19}$$

Here the three-dimensional tensor h^{ij} also equals δ_{ij}, since it is defined to be the reciprocal of g_{ij}. In addition, equation (1.5) shows that $v^i = 0$, so that the ether is at rest in the coordinates x_α, and since equations (1.19) imply that $g^{44} = -1/c^2$, equation (1.3) shows that $\zeta = c$. From equation (1.17) it then follows that $d_e/dx_4 = \partial/\partial x_4$, and equation (1.18) becomes

$$W^i = \frac{-c^2}{\partial x_4'/\partial x_4} \frac{\partial x_4'}{\partial x_i} \tag{1.20}$$

The new time variable x_4' is given by the Lorentz transformation

$$x_4' = \frac{1}{\sqrt{(1 - w^i w^i/c^2)}} \left(x_4 - \frac{w^j x_j}{c^2} \right) \tag{1.21}$$

where w^i are the three parameters of the transformation. Substituting equation (1.21) into equation (1.20) gives

$$W^i = w^i$$

which shows that the ether of the new Lorentz frame is moving with a uniform velocity w^i relative to the ether of the original frame. Thus an observer who uses the time variable x_4' instead of x_4 will consider himself to be fixed in the ether even though he is moving with a velocity w^i relative to the ether of the original coordinates x_α.

In the general case of a curved metric space it is not possible to introduce Cartesian coordinates in which the ether is everywhere at rest, but it is still possible to define a set of Lorentz frames which play a role in the gravitational field similar to the one played by the Lorentz frames of the special theory in the electromagnetic field (Kirkwood, 1970). In each such Lorentz frame there is a uniquely determined time variable, and hence a uniquely determined ether velocity field. The velocity fields associated with two different Lorentz frames will not generally differ by a uniform velocity as they do in the special theory, but it is not difficult to show that in most fields of physical interest the field W^i of equation (1.18) is roughly uniform as long as the parameters of the Lorentz transformation relating the two frames are much less than c. In this respect, the Lorentz frames in a curved space are at least qualitatively similar to those in a flat space.

The above discussion has shown that the introduction of a time variable in a curved metric four-space leads to an interpretation of the four-space as an ether flow in a Riemannian three-space. The general idea that a curved four-space reformulates rather than replaces the ether was known to

Einstein, although he did not go so far as to define an ether velocity field (Einstein, 1934). An explicit description of gravity as an ether velocity field in a Euclidean three-space has been derived from elementary physical principles without the use of tensor calculus (Kirkwood, 1953). Also, an ether-like interpretation of a metric four-space has been given in a series of papers by Jánossy (Jánossy, 1966). In spite of this, most modern theorists describe gravity by a metric four-space and make no reference to its interpretation as an ether flow, sometimes even concluding that Einstein's theory disproves the idea of an ether.

At first glance, it might appear that the formal equivalence of a metric four-space to an ether flow makes the distinction between these two interpretations insignificant. However, this is not the case; in fact, the physical interpretation of the formalism may have a strong influence on the direction of future research. As an illustration, a physicist who thinks in terms of a metric four-space is likely to treat all four coordinates on an equal footing, while one who thinks in terms of an ether flow will tend to give preferential treatment to the time variable. The first physicist might then give his greatest attention to metric tensors which can be diagonalized everywhere, while the second might be more inclined to consider metric fields in which the three-dimensional geometry is nearly Euclidean and the gravitational field is described by the terms $g_{\alpha 4}$, which determine the ether velocity and the velocity of light relative to the ether. As a result, the difference in their interpretations of the same formalism might lead them along very different lines of theoretical investigation.

The interpretation of gravity as an ether flow will be used throughout this paper, and it will be seen to provide a simple and intuitively understandable description of gravity in the classical framework of three dimensions and time and also to suggest some new points of departure, both in gravitational research and in the unification of physics.

2. *Local Determinacy*

Classical physics is based upon the assumption that the motion that enters into fundamental physical laws is motion relative to a primary inertial system which is fixed relative to the fixed stars or to the center of mass of the universe. In the metric four-space discussed above, it was seen that the motion that enters into the laws of mechanics and ray optics is motion relative to the ether, and in this way the ether plays the role of the primary inertial system of classical physics. From a philosophical point of view, this is a great improvement over classical physics, because the motion of a body is then related to the motion of the ether at the point where the body is located rather than to the motion of a remote system such as the fixed stars, and it has always been difficult to accept a direct relation between the motions of widely separated bodies in the absence of any physical connection between them. However, this philosophical advantage was lost in most of the classical ether theories because it was assumed that the motion of the ether itself was determined by laws like those of classical hydrodynamics or elasticity, which compare the velocity of the ether directly to that of the fixed stars. To avoid losing this philosophical advantage, it will be assumed here that *the laws governing the motion of the ether do not refer directly to the motion of any remote point or system of bodies, such as the fixed stars or the center of mass of the universe.* A more detailed discussion of this assumption has been given previously (Kirkwood, 1954), and it has been shown to put a marked limitation on the possible equations of motion of the ether, excluding, for example, most of the equations of motion of the classical ether theories. In this earlier discussion, quantities which do not refer to any remote system such as the fixed stars and equations involving only such quantities have been said to be *determined locally,* and the formulation of locally determined relations has been described in three-dimensional notation in a Euclidean three-space. In this section, a description of locally determined relations will be given in an arbitrary metric four-space, and it will be seen that this description is formally very different from the previous one, even though the underlying physical ideas are the same in both cases.

From equation (1.8) it is seen that the local time $d\tau$ measured by a physical clock in a time interval dx_4 depends only on the motion of the clock relative to the ether and on the function ζ, which is the velocity of light relative to the ether, with the result that *$d\tau$ is a locally determined quantity.* Since geodesics can be defined in terms of the line integral of $d\tau$, they are also determined locally, so that geodesic coordinates constructed with any given point as origin will be determined locally. If a scalar, vector, or tensor quantity has a physical meaning which does not depend on the motion of the fixed stars or of any other remote system, then its partial derivatives with respect to these geodesic coordinates will be determined locally, which implies that the covariant derivative of a locally determined scalar, vector, or tensor is also determined locally. Thus, *tensor relations between locally determined scalars, vectors, and tensors and their covariant derivatives with respect to $g_{\alpha\beta}$ will be determined locally.* The fact that tensor relations of this type are determined locally is not surprising, because tensor relations hold in any coordinate system and thus can involve the motion of the fixed stars or of some other remote system only if this state of motion is explicitly represented in one or more of the tensors involved in the relation. Since the metric tensor and the other tensors considered above do not involve such a preferred state of motion, tensor relations between them would be expected to be determined locally.

Because the curvature tensor can be defined in terms of the commutator of second covariant derivatives, it is clearly one of the tensors that can appear in locally determined relations. However, there are also other tensors that might appear in these relations, and some of these will now be described. The discussion here will be confined to those tensors which depend only on the physical quantities that are involved in the metric tensor $g_{\alpha\beta}$, as they have been described in the previous section. These quantities are the three-dimensional metric on a surface of constant x_4, the ether velocity v^i, and the velocity of light relative to the ether, denoted by ζ.

Considering first the three-dimensional geometry, it is clear that the determination of distances and angles at a given instant of the time variable x_4 does not depend on the motion of the fixed stars or other remote system, so that the three-dimensional metric coefficients g_{ij} are determined locally. This can be stated in a slightly different form by letting $\delta_1 x_\alpha$ and $\delta_2 x_\beta$ be instantaneous increments in the coordinates, arbitrary except that $\delta_1 x_4 = \delta_2 x_4 = 0$, and observing that the three-dimensional geometry can be described in terms of quantities of the form $g_{ij}\delta_1 x_i \delta_2 x_j$, assuming that the special case in which $\delta_1 x_i = \delta_2 x_i$ is included in the analysis. Then the observation made above is simply that all possible quantities of the form $g_{ij}\delta_1 x_i \delta_2 x_j$ are determined locally.

Turning next to the ether velocity field, it is clear that if the motion of the ether is not to be compared with that of some remote system such as the fixed stars, then it is only the motion of the ether relative to nearby objects which is physically meaningful. Since only the motion of the ether itself is being considered here, it can enter into physical laws only as the *relative* motion of nearby points that are fixed in the ether, that is, as the *rate of deformation* of the ether. A point that is fixed in the ether is one that moves so that

$$dx_i = v^i \, dx_4 \tag{2.1}$$

where v^i is given by equation (1.5). If x_α, $x_\alpha + \delta_1 x_\alpha$, and $x_\alpha + \delta_2 x_\alpha$ are the coordinates of three nearby points that are fixed in the ether at a given instant of x_4, then $\delta_1 x_4 = \delta_2 x_4 = 0$, and the rate of deformation of the ether is given by the rates of change of the lengths of the three-dimensional vectors $\delta_1 x_i$ and $\delta_2 x_i$ and of the angle between them. These can be determined from quantities of the form $g_{ij}\delta_1 x_i \delta_2 x_j$, so that the rate of deformation of the ether is determined by the rate of change of $g_{ij}\delta_1 x_i \delta_2 x_j$. This in turn depends on the time derivatives of $\delta_1 x_i$ and $\delta_2 x_j$, which are functions of x_4 only, and on the total time derivative of g_{ij} evaluated at a point fixed in the ether, which can be found from equation (1.17), with the result that

$$\frac{d}{dx_4}(g_{ij}\delta_1 x_i \delta_2 x_j) = \left(\frac{\partial g_{ij}}{\partial x_4} + v^k \frac{\partial g_{ij}}{\partial x_k}\right)\delta_1 x_i \delta_2 x_j + g_{ij}\frac{d\delta_1 x_i}{dx_4}\delta_2 x_j + g_{ij}\delta_1 x_i \frac{d\delta_2 x_j}{dx_4} \tag{2.2}$$

The quantity $d\delta_1 x_i/dx_4$ can be evaluated by noting that $\delta_1 x_i$ is the difference between the coordinates of two points that are fixed in the ether, so that the time derivative of $\delta_1 x_i$ is the difference of the velocities of these two points. Since the velocity of each point is the ether velocity v^i, this implies that

$$\frac{d\delta_1 x_i}{dx_4} = \frac{\partial v^i}{\partial x_k}\delta_1 x_k$$

where k is summed from 1 to 3, since $\delta_1 x_4 = 0$. Multiplying by g_{ij} and noting that v^i is given by equation (1.5) gives

$$g_{ij}\frac{d\delta_1 x_i}{\partial x_4} = g_{ij}\frac{\partial}{\partial x_k}(-h^{i\ell} g_{\ell 4})\,\delta_1 x_k$$
$$= \left(-\frac{\partial g_{j4}}{\partial x_k} - v^i \frac{\partial g_{ij}}{\partial x_k}\right)\delta_1 x_k$$

Using this and a similar relation for $g_{ij}d\delta_2 x_j/dx_4$, equation (2.2) becomes

$$\frac{d}{dx_4}(g_{ij}\,\delta_1 x_i\,\delta_2 x_j) = -2[(ij,4) + v^k(ij,k)]\,\delta_1 x_i\,\delta_2 x_j$$

where (ij,k) is the three-dimensional Christoffel symbol. Using the last two of equations (1.9), this is seen to be

$$\frac{d}{dx_4}(g_{ij}\,\delta_1 x_i\,\delta_2 x_j) = 2\zeta^2 g^{\gamma 4}(\alpha\beta,\gamma)\,\delta_1 x_\alpha\,\delta_2 x_\beta$$

where $(\alpha\beta,\gamma)$ is the four-dimensional Christoffel symbol and the sums over α and β have been extended from 1 to 4 without affecting the result, because $\delta_1 x_4 = \delta_2 x_4 = 0$. Finally, since $\partial x_4/\partial x_\alpha = (0001)$ and all second derivatives of x_4 vanish, it is seen that

$$g^{\gamma 4}(\alpha\beta,\gamma) = -\left[\frac{\partial^2 x_4}{\partial x_\alpha\,\partial x_\beta} - g^{\gamma\delta}(\alpha\beta,\gamma)\frac{\partial x_4}{\partial x_\delta}\right]$$

so that

$$\frac{d}{dx_4}(g_{ij}\,\delta_1 x_i\,\delta_2 x_j) = -2\zeta^2 x_{4;\alpha\beta}\,\delta_1 x_\alpha\,\delta_2 x_\beta \tag{2.3}$$

where $x_{4;\alpha\beta}$ denotes the second covariant derivative of x_4, that is:

$$x_{4;\alpha\beta} \equiv \frac{\partial^2 x_4}{\partial x_\alpha\,\partial x_\beta} - g^{\gamma\delta}(\alpha\beta,\gamma)\frac{\partial x_4}{\partial x_\delta}$$

Turning finally to the function ζ, it is clear that since ζ is the velocity of light relative to the ether it is determined locally and can appear in physical laws. Thus the increment of $1/\zeta^2$ associated with an arbitrary increment Δx_α of the coordinates will also be determined locally. From equation (1.3) this quantity is

$$\Delta\left(\frac{1}{\zeta^2}\right) = -\Delta\left(g^{\alpha\beta}\frac{\partial x_4}{\partial x_\alpha}\frac{\partial x_4}{\partial x_\beta}\right)$$
$$= -2g^{\alpha\beta}\frac{\partial x_4}{\partial x_\alpha}x_{4;\beta}\;\Delta x_\gamma$$

If dx_β is the displacement of a point that is fixed in the ether during an arbitrary time interval dx_4, then $dx_i = v^i dx_4$ and from the last two of equations (1.9) $dx_\beta = -\zeta^2 g^{\beta 4} dx_4$. Thus $g^{\alpha\beta}\,\partial x_4/\partial x_\alpha = g^{\beta 4} = -(1/\zeta^2)\,dx_\beta/dx_4$, and the above relation can be written

$$\Delta\left(\frac{1}{\zeta^2}\right) = \frac{2}{\zeta^2} x_{4;\alpha\beta} \frac{dx_\alpha}{dx_4} \Delta x_\beta \tag{2.4}$$

from which it is clear that $x_{4;\alpha\beta}(dx_\alpha/dx_4)\Delta x_\beta$ is determined locally for any Δx_β and for any dx_4, if dx_α is the displacement of a point fixed in the ether. From equation (2.3) it is seen that $x_{4;\alpha\beta}\delta_1 x_\alpha \delta_2 x_\beta$ is determined locally for any $\delta_1 x_\alpha$ and $\delta_2 x_\beta$ such that $\delta_1 x_4 = \delta_2 x_4 = 0$. Since an arbitrary Δx_α can always be written in the form $\delta x_\alpha + (dx_\alpha/dx_4)\Delta x_4$, where $\delta x_4 = 0$, it follows that for any $\Delta_1 x_\alpha$ and $\Delta_2 x_\beta$ the quantity $x_{4;\alpha\beta}\Delta_1 x_\alpha \Delta_2 x_\beta$ can be written in terms of the locally determined quantities given above, and hence *$x_{4;\alpha\beta}\Delta_1 x_\alpha \Delta_2 x_\beta$ is determined locally for any values of $\Delta_1 x_\alpha$ and $\Delta_2 x_\beta$*. If this quantity is to be expressed in any other coordinates, $\Delta_1 x_\alpha$ and $\Delta_2 x_\beta$ will transform as contravariant four-vectors, with the result that $x_{4;\alpha\beta}$ will transform as a tensor with two covariant indices. As shown above, this means that *tensor relations involving the metric tensor, the curvature tensor (and its covariant derivatives), and covariant derivatives of the time variable x_4 will be determined locally.* However, relations involving the covariant derivatives of x_4 will be determined locally only if the time variable is x_4, and not if any other time variable is used. Thus a relation will generally be determined locally only with respect to one particular time variable, and some discussion is necessary to reconcile the assumption that the laws of physics are determined locally with the requirement that they are Lorentz invariant.

The meaning of Lorentz invariance in the presence of a gravitational field has been discussed previously (Kirkwood, 1970). It has been shown that if the metric coefficients $g_{\alpha\beta}$ are written in one Lorentz frame in terms of a set of functions β, V, t, λ_u, and μ_u in the form

$$g_{\alpha\beta} = \sigma_{\alpha\beta} - 2V\frac{\partial t}{\partial x_\alpha}\frac{\partial t}{\partial x_\beta} + \sum_{n=1}^{N} \lambda_n \frac{\partial \mu_n}{\partial x_\alpha}\frac{\partial \mu_n}{\partial x_\beta} \tag{2.5}$$

where N can be as large as desired and where $\sigma_{\alpha\beta}$ is given by

$$\begin{aligned} \sigma_{ij} &= \delta_{ij} \\ \sigma_{i4} &= -\frac{\partial \beta}{\partial x_i} \\ \sigma_{44} &= -c^2 - 2\frac{\partial \beta}{\partial x_4} \end{aligned} \tag{2.6}$$

then the coordinates x_α' of a new Lorentz frame are defined so that x_i' and $x_4' + \beta/c^2$ are related to x_i and $x_4 + \beta/c^2$ by the usual Lorentz transformation of the special theory. The functional form of the metric coefficients given by equations (2.5) and (2.6) is the same in all Lorentz frames when β, V, t, λ_n, and μ_n are treated as invariant functions. With this formulation of

Lorentz invariance, it is clear that *tensor relations involving the metric tensor, the curvature tensor, and the functions* β, V, t, λ_n, *and* μ_n *and their covariant derivatives will have the same functional form in all Lorentz frames when they are expressed in terms of* β, V, t, λ_n, *and* μ_n *and their partial derivatives*, and in this sense these relations will be Lorentz invariant.

Comparing these Lorentz invariant relations with the locally determined relations found above, it is seen that tensor relations which involve the field quantities only through the metric tensor and the curvature tensor (including its covariant derivatives) will be both Lorentz invariant and determined locally. Relations of this type were the ones considered by Einstein in his formulation of gravity. Since they do not involve the coordinate x_4 explicitly, they will be determined locally with respect to *any* time variable, and hence will be determined locally in *any* Lorentz frame. There is no doubt that this property gives these relations a certain formal and philosophical beauty, but there is no philosophical *necessity* for requiring that the laws of physics must be determined locally in every Lorentz frame, and it is possible that Einstein's theory may have been too restrictive in this respect.

In most gravitational fields that are of physical interest, the coordinates x_α in one Lorentz frame can be chosen so that $x_4 = t$, and in this frame the terms involving λ_n and μ_n in equation (2.5) can be neglected. The three-dimensional geometry is then Euclidean and the function V is the Newtonian gravitational potential. In such fields, tensor relations involving the metric and curvature tensors and the covariant derivatives of t are Lorentz invariant and are also determined locally in the one Lorentz frame in which $x_4 = t$. Furthermore, in an arbitrary metric field, laws of this type will be both Lorentz invariant and determined locally with respect to the invariant time function t, and this is all that is really required by the philosophical considerations discussed above. Therefore, it will be assumed here that *the laws of physics can be written as tensor relations in which space, time, and gravity appear only through the metric tensor, the curvature tensor (and its covariant derivatives), and the covariant derivatives of the invariant time function t.* This assumption puts a severe restriction on the possible equations that might describe the gravitational field but is less restrictive than the assumptions of Einstein's theory, which do not include the possible existence of an invariant time function t.

3. *The field Equations*

The considerations of the previous section are not sufficient by themselves to determine the gravitational field uniquely, and some additional facts are needed to specify the field equations. Without doubt, the facts which are established with the greatest certainty are those described by Newton's theory of gravity, and the most straightforward procedure for finding field equations that agree with the results of the previous section is to formulate Newtonian theory in the framework of a metric four-space. This has been done in a previous paper (Kirkwood, 1972), where it has been shown that the Newtonian fields are solutions of the equation

$$\rho_{\alpha\beta} = -4\pi K\mu \frac{\partial t}{\partial x_\alpha}\frac{\partial t}{\partial x_\beta} \tag{3.1}$$

where K is the gravitational constant, μ the mass density, t the invariant time function, and

$$\rho_{\alpha\beta} \equiv (g^{\gamma\delta} + \alpha^2 t^\gamma t^\delta)[R_{\alpha\gamma\delta\beta} + \alpha^2(t_{;\gamma\delta}t_{;\alpha\beta} - t_{;\gamma\beta}t_{;\alpha\delta})] \tag{3.2}$$

Here $R_{\alpha\gamma\delta\beta}$ is the curvature tensor determined from $g_{\alpha\beta}$, t^α is defined by

$$t^\alpha \equiv g^{\alpha\beta}\frac{\partial t}{\partial x_\beta} \tag{3.3}$$

and the function α is the velocity of light relative to the ether in coordinates in which $x_4 = t$. Thus α is the function ζ of equation (1.3) when the time variable is t, and hence is defined so that

$$g^{\alpha\beta}\frac{\partial t}{\partial x_\alpha}\frac{\partial t}{\partial x_\beta} \equiv -\frac{1}{\alpha^2} \tag{3.4}$$

Equation (3.1) is a complete and exact description of Newtonian gravitational fields, including the fact that the three-dimensional geometry at one instant of t is Euclidean. When $x_4 = t$ and the spatial coordinates x_i are Cartesian, the solutions of equation (3.1) are given by equation (2.5) with $\lambda_n = 0$, that is, by

$$\begin{aligned} g_{ij} &= \delta_{ij} \\ g_{i4} &= -\frac{\partial\beta}{\partial x_i} \\ g_{44} &= -2V - 2\frac{\partial\beta}{\partial x_4} - c^2 \end{aligned} \tag{3.5}$$

where V is a solution of Poisson's equation. It has been shown previously (Kirkwood, 1970) that if $V = -KM/r$ and $\beta = -(8KMr)^{1/2}$, equations (3.5) become the Schwarzschild field, and hence they yield all of the relativistic corrections to Newton's theory that have been verified in the Schwarzschild field, provided that $d\tau$ is given by equation (1.1) and the orbits of particles and light rays are assumed to be geodesics.

Since $\rho_{\alpha\beta}$ is a tensor which involves only the metric tensor, the curvature tensor, and the covariant derivatives of t, equation (3.1) is determined locally with respect to the time variable t. It would also be Lorentz invariant if $K\mu$ were invariant under the Lorentz transformation, but this would conflict with the known transformation properties of μ as they are given in the special theory. To overcome this difficulty, it has been assumed previously that the right side of equation (3.1) is only an approximate form of a tensor which involves only the metric tensor and the stress-energy-momentum tensor of the special theory. The three-dimensional stresses have been specified in such a way as to maintain the Euclidean nature of three-space at one instant of t, and the weak-field approximation of the field equation has been made closely analogous to classical electrodynamics, which is very desirable if gravity and electromagnetism are to be unified. It is then found that equation (3.1) is replaced by

$$\rho_{\alpha\beta} = -\frac{2\pi K}{c^4}(P_{\alpha\beta} - \tfrac{1}{2}Pg_{\alpha\beta}) \tag{3.6}$$

where $P_{\alpha\beta}$ is the stress-energy-momentum tensor associated with ponderable matter and $P \equiv g^{\alpha\beta}P_{\alpha\beta}$. Since $P_{\alpha\beta} = 0$ where $\mu = 0$, the *external* Newtonian fields, including the external Schwarzschild field, still satisfy equation (3.6). Furthermore, equation (3.6) is determined locally with respect to the time variable t, and the mass density enters into it only as P^{44} and hence transforms as required by the special theory.

Although equation (3.6) is formally ten equations, it has been assumed that six of these equations merely define the three-dimensional stress in such a way that the three-dimensional geometry at one instant of t is Euclidean. Thus it is possible to choose coordinates so that $x_4 = t$ and the spatial coordinates x_i are Cartesian, and in these coordinates equation (3.6) reduces to four field equations involving the coefficients $g_{\alpha 4}$. These four equations have been shown to be

$$\begin{aligned} \frac{\partial}{\partial x_j}\left(\frac{\partial g_{j4}}{\partial x_i} - \frac{\partial g_{i4}}{\partial x_j}\right) &= \frac{4\pi K g}{c^4}\mu(u^i + g_{i4}) \\ \frac{1}{2}\frac{\partial^2 g_{44}}{\partial x_i \partial x_i} - \frac{\partial^2 g_{i4}}{\partial x_4 \partial x_i} - \frac{1}{4}\left(\frac{\partial g_{i4}}{\partial x_j} - \frac{\partial g_{j4}}{\partial x_i}\right)\left(\frac{\partial g_{i4}}{\partial x_j} - \frac{\partial g_{j4}}{\partial x_i}\right) & \\ = -\frac{4\pi K g}{c^4}\mu[g + (u^i + g_{i4})g_{i4}] & \end{aligned} \tag{3.7}$$

Here u^i is the velocity of ponderable matter, defined so that $P^{i4} = \mu u^i$, and $g = g_{44} - v^2 = -\alpha^2$ from equation (1.7), noting that $\zeta = \alpha$ and that $g_{ij} = \delta_{ij}$ and $h = 1$ in these coordinates, and denoting $v^i v^i$ by v^2.

From the results of the first section of this paper, this Lorentz-invariant extension of Newtonian theory is readily interpreted in terms of an ether flow. In the coordinates of equations (3.7), equation (1.5) shows that $g_{i4} = -v^i$, and g_{44} is given by $-\alpha^2 + v^2$, so that equations (3.7) can be written in three-dimensional vector notation in the form

$$\nabla x \nabla x \mathbf{v} = \frac{4\pi K \alpha^2}{c^4}\mu(\mathbf{u} - \mathbf{v}) \tag{3.8}$$

$$\tfrac{1}{2}\nabla^2(-\alpha^2 + v^2) + \frac{\partial}{\partial x_4}\nabla . v - \tfrac{1}{2}(\nabla x v)^2 = -\frac{4\pi K \alpha^2}{c^4}\mu[\alpha^2 + (\mathbf{u} - \mathbf{v}).\mathbf{v}]$$

The second of these equations can be rewritten by replacing $(4\pi K\alpha^2/c^4) \times \mu(\mathbf{u} - \mathbf{v})$ by use of the first equation and making use of the vector identity

$$\tfrac{1}{2}\nabla^2 v^2 - \tfrac{1}{2}(\nabla x v)^2 + \mathbf{v}.\nabla x \nabla x \mathbf{v} = \boldsymbol{\phi} : \boldsymbol{\phi} + \mathbf{v}.\nabla(\nabla . \mathbf{v})$$

where ϕ is the classical rate-of-strain tensor, whose components are

$$\phi_{ij} \equiv \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{3.9}$$

and where $\phi:\phi \equiv \phi_{ij}\phi_{ij}$. Equations (3.8) then become

$$\begin{aligned} \nabla x \nabla x \mathbf{v} &= \frac{4\pi K\alpha^2}{c^4}\mu(\mathbf{u}-\mathbf{v}) \\ -\tfrac{1}{2}\nabla^2\alpha^2 + \frac{d_e}{dx_4}(\nabla . v) + \phi:\phi &= -\frac{4\pi K\alpha^4}{c^4}\mu \end{aligned} \tag{3.10}$$

where d_e/dx_4 is the total time derivative at a point fixed in the ether, defined by equation (1.17). In the nonrelativistic limit in which $c \to \infty$, if it is assumed that $\alpha/c \to 1$, equations (3.10) become

$$\begin{aligned} \nabla x \nabla x \mathbf{v} &= 0 \\ -\tfrac{1}{2}\nabla^2\alpha^2 + \frac{d_e}{dx_4}(\nabla . \mathbf{v}) + \phi:\phi &= -4\pi K\mu \end{aligned} \tag{3.11}$$

When α is a constant, these relations reduce to the field equations that have been obtained previously directly from Newtonian theory (Kirkwood, 1954), as is to be expected from the way in which they were derived.

In general, equations (3.7) or (3.8) do not determine the metric coefficients uniquely, and this lack of uniqueness can be exhibited explicitly in regions in which $\mu = 0$. Here, the right sides of equations (3.7) vanish, with the result that if g_{i4} and g_{44} are one set of solutions of equations (3.7) then $g_{i4} + \partial\chi/\partial x_i$ and $g_{44} + 2\partial\chi/\partial x_4$ are easily seen to be another set of solutions, where χ is an arbitrary function. In this way the free-space equations admit of a gauge invariance similar to the one found in electromagnetism. However, replacing g_{i4} and g_{44} by $g_{i4} + \partial\chi/\partial x_i$ and $g_{44} + 2\partial\chi/\partial x_4$ replaces $-c^2 d\tau^2$ of equation (1.1) by $-c^2 d\tau^2 + 2dx_4 d\chi$ and hence affects the value of $d\tau$, so that the function χ has physical meaning and cannot be specified arbitrarily, as is done in the case of the electromagnetic gauge. For this reason, the additional relation that is required to determine the field uniquely is not arbitrary and must be determined from observation. It will now be shown that this additional relation has no effect on the predictions of the theory in the nonrelativistic limit as $c \to \infty$, and hence it must be determined from observations of the relativistic corrections to Newtonian theory.

In the nonrelativistic approximation, the field satisfies equations (3.11), the first of which leads to solutions of the form $\mathbf{v} = \nabla\beta$. For these fields the second of equations (3.8) becomes, in the limit as $c \to \infty$,

$$\nabla^2\left[\frac{\alpha^2 - (\nabla\beta)^2}{2} - \frac{\partial\beta}{\partial x_4}\right] = 4\pi K\mu$$

Assuming that $\alpha \to \infty$ as $c \to \infty$ in such a way that $\alpha^2 - c^2$ is finite, this is seen to be Poisson's equation for the Newtonian potential

$$V = \frac{\alpha^2 - c^2}{2} - \frac{(\nabla\beta)^2}{2} - \frac{\partial\beta}{\partial x_4} \tag{3.12}$$

where the term $-c^2/2$ is added so that $V = 0$ in flat space, where $\beta = 0$ and $\alpha = c$. Since $x_4 = t$ and α is the function ζ of equation (1.8), equation (1.8) shows that $d\tau \to dx_4$ as α and c approach infinity in such a way that $\alpha/c \to 1$. Thus, in the nonrelativistic limit, a moving clock will measure the time coordinate x_4. Under these circumstances the geodesic equation becomes (Kirkwood, 1972):

$$\frac{d^2x_i}{dx_4{}^2} = \left(\frac{\partial g_{j4}}{\partial x_i} - \frac{\partial g_{i4}}{\partial x_j}\right)\frac{dx_j}{dx_4} - \frac{\partial g_{i4}}{\partial x_4} + \frac{1}{2}\frac{\partial g_{44}}{\partial x_i}$$

Since $g_{i4} = -v^i = -\partial\beta/\partial x_i$, the first term on the right side vanishes, and this equation is

$$\frac{d^2x_i}{dx_4{}^2} = -\frac{\partial}{\partial x_i}\left(-\frac{\partial\beta}{\partial x_4} - \tfrac{1}{2}g_{44}\right)$$

where, as in equations (3.8), $g_{44} = -\alpha^2 + v^2 = -\alpha^2 + (\nabla\beta)^2$. This shows that the acceleration of a moving body or light ray is equal to the negative gradient of the Newtonian potential V given by equation (3.12). Thus bodies move as predicted by Newtonian theory and light travels instantaneously in straight lines, because a finite acceleration does not alter the infinite velocity of light. The motion of a body or a light ray and the time measured by a physical clock are completely determined in the nonrelativistic approximation by the potential function V. Thus, an observation that is accurate only to the nonrelativistic approximation depends only on the value of V and can never determine either α or β, except to the extent that α and β must yield the proper value of V when they are used in equation (3.12). Replacing g_{i4} by $g_{i4} + \partial\chi/\partial x_i$ and g_{44} by $g_{44} + 2\partial\chi/\partial x_4$ is equivalent to replacing β by $\beta - \chi$ in the nonrelativistic approximation, because here $g_{i4} = -\partial\beta/\partial x_i$ and $g_{44} = -\alpha^2 + (\nabla\beta)^2$, which, from equation (3.12), shows that $g_{44} = -c^2 - 2V - 2\partial\beta/\partial x_4$. Because β cannot be determined from measurements accurate only to the nonrelativistic approximation, χ cannot be determined either.

Thus the only evidence concerning gravity that might suggest the desired additional relation is to be found in the relativistic corrections to Newtonian theory. The existence of gravitational radiation is one such correction, and it is not difficult to add an additional relation to the field equations in such a way that the theory will predict radiation. However, so little observational information is available concerning radiated fields that the additional relation is not determined uniquely, and the motivation for any such procedure is very weak at present. The only other verifiable corrections to Newtonian theory are those that have been found in the static, spherically symmetric field about a single mass, and this field will now be investigated in detail, assuming that equations (3.7) are an exact, although incomplete, description of the field.

4. *The Spherically Symmetric Field*

The static, spherically symmetric solution of equations (3.7) can be found in the external region where $\mu = 0$ by noting that $x_4 = t$ and $g_{ij} = \delta_{ij}$, so that $g_{i4} = -v^i$, and that the spherical symmetry implies that the ether velocity $\mathbf{v}$ must be radial and have a magnitude depending only on the distance r from the center of symmetry. Therefore, $\nabla x \mathbf{v}$ vanishes and a function β exists such that $\mathbf{v} = \nabla\beta$, so that the first of equations (3.7) is satisfied identically in the region where $\mu = 0$. Because the field is static, the quantity $\partial^2 g_{i4}/\partial x_4 \partial x_i$ vanishes, and the second of equations (3.7) reduces to $\nabla^2 g_{44} = 0$ in this region. If g_{44} is to approach its flat-space value of $-c^2$ as $r \to \infty$, this implies that $g_{44} = 2KM/r - c^2$, where M is a parameter which differs from the total mass producing the field only by relativistic corrections which are too small to be significant in the solar field (Kirkwood, 1972). Since $g_{44} = -\alpha^2 + v^2$, where $v^2 = (\nabla\beta)^2$, it follows that

$$\alpha^2 = c^2 + v^2 - \frac{2KM}{r} \tag{4.1}$$

Thus the exact form of equations (3.7) determines only $\alpha^2 - v^2$, which is the same as $\alpha^2 - (\nabla\beta)^2$, and does not determine α and β separately, just as in the case of the nonrelativistic approximation described in the previous section.

The gravitational red shift can be found from equation (1.8). When $x_4 = t$, $g_{ij} = \delta_{ij}$, and $\zeta = \alpha$, equation (1.8) becomes

$$-c^2\,d\tau^2 = (dx_i - v^i\,dt)(dx_i - v^i\,dt) - \alpha^2\,dt^2 \tag{4.2}$$

For a clock that is fixed in the coordinates x_i, $dx_i = 0$, and equation (4.2) becomes

$$-c^2\,d\tau^2 = (v^2 - \alpha^2)\,dt^2$$

Using equation (4.1), this gives

$$d\tau = \sqrt{\left(1 - \frac{2KM}{c^2 r}\right)}\,dt$$

This is exactly Einstein's equation for the gravitational red shift in the Schwarzschild field, and it is apparent that this relation is independent of the way in which α and β are specified as long as equation (4.1) is satisfied, and therefore that measurements of the red shift in the spherically symmetric field will give no information about the values of α and β separately.

The orbital equations are found from $\delta \int d\tau = 0$, where $d\tau$ is given by equation (4.2), and are the classical Euler-Lagrange equations derived from the variation principle $\delta \int L\,dt = 0$, where

$$L \equiv \frac{1}{c}\sqrt{\left[\alpha^2 - \left(\frac{dx_i}{dt} - v^i\right)\left(\frac{dx_i}{dt} - v^i\right)\right]} \tag{4.3}$$

If r and θ are polar coordinates in the plane of the orbit and their time derivatives are denoted by $\dot{r}$ and $\dot{\theta}$, then

$$L = \frac{1}{c}\sqrt{(\alpha^2 - v^2 - \dot{r}^2 - r^2\dot{\theta}^2 + 2v\dot{r})} \tag{4.4}$$

The spherical symmetry implies that α and v are functions of r only, so that L does not depend explicitly on either θ or t, and hence the equations of motion have energy and angular momentum integrals given by

$$\begin{aligned} h &= \frac{\partial L}{\partial \dot{r}}\dot{r} + \frac{\partial L}{\partial \dot{\theta}}\dot{\theta} - L = \frac{1}{c^2 L}(-\alpha^2 + v^2 - v\dot{r}) \\ k &= \frac{\partial L}{\partial \dot{\theta}} = -\frac{r^2}{c^2 L}\dot{\theta} \end{aligned} \tag{4.5}$$

Since α is the velocity of light relative to the ether, it is clear from equation (4.3) that $L = 0$ along the path of a ray of light. Equations (4.5) then show that h and k are both infinite for such an orbit while the ratio of h/k is finite. Thus the path of a ray of light can be found as a limiting case of equations (4.5) by letting h and k approach infinity in such a way that h/k is finite, and there is no need for a separate analysis of the orbits of light rays.

To find the orbital equation, it is convenient to let $r' \equiv dr/d\theta = \dot{r}/\dot{\theta}$ and to write L, h, and k in terms of r' and $\dot{\theta}$:

$$\begin{aligned} L &= \frac{1}{c}\sqrt{(\alpha^2 - v^2 - (r'^2 + r^2)\dot{\theta}^2 + 2vr'\dot{\theta})} \\ h &= \frac{1}{c^2 L}(-\alpha^2 + v^2 - vr'\dot{\theta}) \\ k &= -\frac{r^2}{c^2 L}\dot{\theta} \end{aligned} \tag{4.6}$$

Then $\dot{\theta}$ can be found from the ratio h/k, and is

$$\dot{\theta} = \frac{\alpha^2 - v^2}{(h/k)r^2 - vr'}$$

Using this value of $\dot{\theta}$ in the first of equations (4.6) and using the resulting expression for L as well as the one for $\dot{\theta}$ in either the second or third of these equations leads to the orbital equation

$$\frac{\alpha^2}{r^2}r'^2 + \left(1 + \frac{r^2}{k^2 c^2}\right)(\alpha^2 - v^2) - r^2\frac{h^2}{k^2} = 0 \tag{4.7}$$

Letting $u \equiv 1/r$ and $u' \equiv du/d\theta$, and noting from equation (4.1) that $\alpha^2 - v^2 = c^2 - 2KM/r$ gives

$$\alpha^2 u'^2 = -\frac{1 - h^2}{k^2} + \frac{2KM}{c^2 k^2}u - c^2 u^2 + 2KMu^3 \tag{4.8}$$

If $\alpha = c$ this is Einstein's equation for the orbit of a planet, and if $\alpha = c$ and h and k both approach infinity in such a way that h/k is finite, this is Einstein's equation for the orbit of a light ray. This is to be expected, because if $\alpha = c$ the magnitude of v^2 is found from equation (4.1) to be $2KM/r$, and this velocity field has been shown previously to lead exactly to the Schwarzschild field of Einstein's theory (Kirkwood, 1953).

If α is not constant but is instead an arbitrary function of r, the orbital equation for either a particle or a light ray differs from the equation given by Einstein's theory only in that cu' is replaced by $\alpha u'$. If a new independent variable θ^* is defined along the orbit so that $d\theta^* = (c/\alpha)d\theta$ and $\theta^* = 0$, where $\theta = 0$, then the orbital equation in terms of θ^* is exactly Einstein's orbital equation. As a result, the value of r that would have occurred in Einstein's theory at the azimuth angle θ^* will now occur at the actual azimuth angle θ, and the orbits can be found from those of Einstein's theory when θ^* is known as a function of θ.

The advance of the perihelion of a planet can be determined by noting that most planetary orbits are roughly circular and that α is assumed to depend only on r, so that α will be nearly constant along the orbit. Then the relation $d\theta^* = (c/\alpha)d\theta$ can be integrated to give $\theta^* = (c/\alpha)\theta$. The perihelion will occur at the point where θ^* is greater than 2π by the amount $6\pi KM/c^2 r$, as predicted by Einstein. At this point

$$\begin{aligned}\theta &= (\alpha/c)\,\theta^* = [1 + (\alpha - c)/c]\,(2\pi + 6\pi KM/c^2 r)\\ &\approx 2\pi + 6\pi KM/c^2 r + 2\pi(\alpha - c)/c\end{aligned}$$

so the actual advance of the perihelion exceeds that of Einstein's theory by the amount $2\pi(\alpha - c)/c$. If the ether were at rest, so that $v = 0$, then the entire Newtonian potential would arise from the variation of α, and it would follow from equation (4.1) that $\alpha^2 - c^2 = -2KM/r$, or that $(\alpha - c)/c \approx -KM/c^2 r$. Thus the perihelion would advance by the amount given in Einstein's theory plus the additional amount $-2\pi KM/c^2 r$, or by the total amount $4\pi KM/c^2 r$, which is only two-thirds of the value predicted by Einstein's theory. If it is assumed that the advance of the perihelion arises entirely from relativistic effects and not from other causes such as the oblateness of the sun, then the observed advance of Mercury is much closer to the value obtained by Einstein, which corresponds to $\alpha = c$, than to two-thirds of this amount, which is the value obtained by assuming that the entire Newtonian potential arises from the variation of α and that $v = 0$. To this extent, the observational evidence suggests that $\alpha = c$ in the spherically symmetric field.

In the case of a light ray, the orbit is nearly a straight line. The relation between θ^* and θ can be found by integrating $d\theta^*$ along the path of the ray, assuming that it starts from the direction $\theta = \theta^* = 0$ and that $d\theta = (\alpha/c)d\theta^*$, so that

$$\theta = \theta^* + \int_0^{\theta^*} (\alpha - c)/c\, d\theta^*$$

In a field in which $v = 0$ and the Newtonian potential arises entirely from variations of α, it was shown above that $(\alpha - c)/c \approx -KM/c^2 r$. If R is the distance of closest approach of the ray to the center of symmetry, then $r \approx R/\sin\theta$, so that $(\alpha - c)/c \approx -KM \sin\theta/c^2 R$, and

$$\theta \approx \theta^* - \int_0^{\theta^*} (KM \sin\theta/c^2 R)\, d\theta^*$$

The integral in this relation represents only a small correction to θ^*, and it is sufficiently accurate to evaluate the integral by assuming that $\theta \approx \theta^*$ in the integrand. With this approximation, it is found that

$$\theta = \theta^* + (KM/c^2 R)(\cos\theta^* - 1)$$

and this implies that $\theta \approx \theta^* - 2KM/c^2 R$ in the direction in which the ray leaves the center of symmetry. According to Einstein's theory, the value of θ^* in this direction is $\pi + 4KM/c^2 R$, so that θ is approximately $\pi + 4KM/c^2 R - 2KM/c^2 R$, and the assumption that $v = 0$ and that the Newtonian potential arises entirely from the variation of α decreases the deflection to one-half of the value predicted by Einstein's theory. Although this deflection is difficult to measure with precision, the results of observation tend to be closer to Einstein's prediction than to one-half of this value, and this again suggests that it is more accurate to assume that $\alpha = c$ than that $v = 0$ and that the entire Newtonian potential arises from the variation of α.

The discussion above suggests that $\alpha = c$ in the spherically symmetric field about a single mass, and this naturally raises the question of whether α has the constant value c in all fields, which would then provide the additional relation needed to augment the field equations of the previous section. Unfortunately, however, if α is always equal to c, the theory outlined above does not lead to gravitational radiation and thus does not agree with the recent observations which suggest that such radiation exists. As a result, it appears that the desired additional relation cannot be determined from the presently known facts concerning gravity. However, the very close analogy that has been shown previously to exist between the weak-field approximation of gravity and the classical electromagnetic field offers a suggestion which may throw some light on this situation. In a static or nearly static gravitational field, the analog of the electrostatic field is the three-dimensional gradient of $g_{44}/2$ (Kirkwood, 1972). Since the flat-space value of g_{44} is $-c^2$, it follows that the gravitational analog of the electrostatic potential is $-(g_{44} + c^2)/2$. Noting that $\alpha^2 = -g_{44} + g_{i4}g_{i4}$, where $g_{i4}g_{i4}$ is quadratic in the field quantities and hence is negligible in the weak-field approximation, the gravitational analog of the electrostatic potential is seen to be $(\alpha^2 - c^2)/2$, and the condition that $\alpha = c$ is the analog of the condition that the electrostatic potential vanishes. It is commonly assumed that electrostatic fields are not of great importance in outer space, and this might suggest by analogy that $\alpha = c$ in all nearly static gravitational fields of astronomical dimensions. However, it is certain that not *all* electrostatic fields vanish, and the gravitational analog of electromagnetism would therefore suggest that α cannot be equal to c in every conceivable gravitational field. This agrees with the conclusion that $\alpha = c$ in the solar field and still does not necessarily conflict with the existence of gravitational radiation. However, reasoning about gravity through its analogy to electromagnetism is a very speculative procedure at present, and this suggestion must await confirmation either from additional observational evidence or from the unification of gravity with the rest of physics.

5. *The Observer's Coordinates*

Because the geometry is Euclidean in the three-dimensional space determined by a given value of the invariant time function t, it has been possible to simplify the formal description of gravity by choosing the time-like coordinate x_4 to equal t and the three spatial coordinates x_i to be Cartesian. Choosing the time-like coordinate to equal t is also very natural from a philosophical point of view, because the theory is then determined locally and does not refer directly to the motion of any remote system such as the fixed stars. However, the formal and philosophical simplicity of the coordinates x_α does not necessarily imply that they are the ones that will be measured directly by physical rods and by physical clocks which are synchronized by light rays or other physical means, and it cannot be concluded that the coordinates x_α are those of one observer of the special theory. Since most observations concerning gravity are made on an astronomical scale, where it is not possible to construct a physical coordinate lattice, this fact is not usually important in gravitational theory. Moreover, in most laboratory experiments, the laboratory itself is small enough so that coordinates can be defined in which the gravitational field within the laboratory is very nearly uniform. When this is the case, it is only necessary to choose the coordinates so that they reduce the metric tensor to its flat-space value at one point, and they will then represent the observer's coordinates of the special theory throughout the laboratory. However, there are some circumstances which involve phenomena other than gravity in regions whose dimensions are comparable to those of the gravitational field, and in these circumstances it is not immediately apparent how the observer's coordinates of the special theory are related to the coordinates x_α used above.

Consider, for example, the usual analysis of the magnetic field of the earth, which is assumed to obey Maxwell's equations in non-rotating coordinates that are fixed relative to the center of the earth. These coordinates are taken to be those of one Lorentz frame of the special theory, and the metric coefficients are assumed to have the flat-space values given by equations (1.19). If gravitational phenomena are involved in the calculations, gravity is taken into account by simply superimposing Newtonian theory on the flat four-space of the special theory. Although this procedure is sufficiently accurate to describe the observed facts, it is theoretically inconsistent with a metric-space description of gravity, in which there are *no* coordinates that reduce the metric to its flat-space value everywhere. Thus, the above procedure can be justified theoretically only if observer's coordinates can be defined in which the metric tensor is at least very nearly equal to its flat-space value everywhere. If such coordinates can be found, they can be used without affecting the predictions of the gravitational theory, because these are independent of the coordinatization. However, in such coordinates the electromagnetic field equations will take essentially their flat-space form, which is the form that is assumed to describe the earth's magnetic field.

It is clear that if the Cartesian coordinates x_i of the gravitational theory are assumed to be fixed relative to the center of the earth and non-rotating relative to the stars, they will be very much like the spatial coordinates that are usually used to describe the magnetic field of the earth. In particular, the three-dimensional metric will be δ_{ij}, and in most physically important fields the velocity of light α will be very nearly equal to c, so the four-dimensional metric tensor will differ from the flat-space metric only in that the ether velocity does not vanish. This suggests that if a new time variable can be introduced in such a way that the new ether velocity field is very small, then the three spatial coordinates x_i and the new time variable will come very close to reducing the metric tensor to its flat-space value, and hence these new coordinates can reasonably be interpreted to be the observer's coordinates of the special theory. If the observer's coordinates are denoted by $\bar{x}_\alpha$, this implies that $\bar{x}_i = x_i$, and that $\bar{x}_4$ should be defined to minimize the ether velocity field.

It has been shown previously that the introduction of a new time variable $\bar{x}_4$ defines a new ether velocity such that points fixed in this new ether will move with velocity W^i relative to the ether of the original coordinates, where W^i is given by equation (1.18) with x_4' replaced by $\bar{x}_4$. Since $x_4 = t$ and the spatial coordinates are Cartesian, the quantities h^{ij} of equation (1.18) equal δ_{ij} and $\zeta = \alpha$, so that equation (1.18) becomes

$$W^i = -\frac{\alpha^2}{d_e\bar{x}_4/dx_4}\frac{\partial \bar{x}_4}{\partial x_i} \tag{5.1}$$

If $\bar{x}_4$ is to be defined so that a point fixed in the ether of the coordinates $\bar{x}_\alpha$ is very nearly at rest in the coordinates x_i, then W^i must be very nearly equal to $-v^i$, where v^i is the ether velocity in the coordinates x_α. Since all known fields are nearly Newtonian, equation (1.5) and equations (3.5) show that $v^i = -g_{i4} \approx \partial\beta/\partial x_i$, and equation (5.1) plus the condition that $W^i \approx -v^i$ implies that

$$-\frac{\alpha^2}{d_e\bar{x}_4/dx_4}\frac{\partial \bar{x}_4}{\partial \bar{x}_i} \approx -\frac{\partial\beta}{\partial x_i} \tag{5.2}$$

This equation can be satisfied *exactly* only if $-\alpha^2/(d_e\bar{x}_4/dx_4)$ is a function of $\bar{x}_4$ alone, but it can be satisfied *approximately* if it is assumed that $\alpha \approx c$

and that $\bar{x}_4$ is not greatly different from x_4, so that $d_e\bar{x}_4/dx_4 \approx 1$, in which case equation (5.2) becomes approximately

$$\frac{\partial}{\partial x_i}(c^2\bar{x}_4 - \beta) = 0$$

This is satisfied if $c^2\bar{x}_4 - \beta$ is a function of x_4 only, and this function will be assumed to be c^2x_4, as this makes $\bar{x}_4$ very nearly equal to x_4. Then the observer's coordinates $\bar{x}_\alpha$ of one Lorentz frame of the special theory will be given by

$$\begin{aligned} \bar{x}_i &= x_i \\ \bar{x}_4 &= x_4 + \beta/c^2 \end{aligned} \tag{5.3}$$

In the nonrelativistic limit as $c \to \infty$, it is seen that $\bar{x}_\alpha \to x_\alpha$.

Admittedly, the argument leading to equations (5.3) is based entirely on physical intuition arising from the interpretation of gravity as an ether flow, but it is confirmed by one additional fact which suggests that it is at least a reasonable working hypothesis for future investigations. This is the fact demonstrated previously (Kirkwood, 1970) that if $x_4 = t$ and x_i are Cartesian coordinates, then the observer's coordinates $\bar{x}_\alpha$ defined by equations (5.3) (denoted previously by X_α) have the property that the observer's coordinates in two different Lorentz frames are related to each other by the usual Lorentz transformation of the special theory. In this respect, the coordinates $\bar{x}_\alpha$ are *exactly* like the usual coordinates of the special theory, and this provides additional motivation for the assumption that *the natural coordinates x_α of gravitational theory are related to the observer's coordinates $\bar{x}_\alpha$ of the special theory by equations (5.3)*. This conclusion is very important to any attempt to unify physics, because gravity is most naturally described in the coordinates x_α while all of the rest of physics is described in the observer's coordinates $\bar{x}_\alpha$ of the special theory, and gravity can be compared accurately with the rest of physics only if the relation between x_α and $\bar{x}_\alpha$ is known.

6. *Conclusions*

It has been shown that any metric four-space can be interpreted as an ether-flow in a Riemannian three-space when a time-like coordinate has been introduced. This interpretation makes it possible to visualize the four-space in a classical framework of three dimensions and time, and the physical intuition developed in everyday life can be applied directly to gravitational phenomena.

In particular, one conclusion that is suggested by this interpretation of gravity is that the laws of physics should depend only upon motion relative to the ether or on the relative motion of nearby points that are fixed in the ether. This restriction is a philosophical improvement over classical physics and over the classical ether theories, because physics is then described without direct reference to any remote system such as the fixed stars. Analytically, this restriction will be satisfied if the laws of physics can be written as tensor relations which involve the gravitational field only through the metric tensor, the curvature tensor and its covariant derivatives, and the covariant derivatives of the time variable. A previous investigation of the meaning of Lorentz invariance in the presence of a gravitational field has suggested the existence of an invariant time-like function t similar to the classical Newtonian time variable, and physical laws that are determined locally with respect to this time variable will also be Lorentz invariant. Therefore, it has been assumed that physics can be described by tensor relations which involve gravity only through the metric tensor, the curvature tensor and its covariant derivatives, and the covariant derivatives of the invariant time function t. This description of gravity is less restrictive than Einstein's because it includes the possibility that the invariant time function t may appear in physical laws.

The interpretation of gravity as an ether flow suggests an approach to the formulation of gravitational field equations which is very different from the one usually adopted in a metric-space description of gravity. Where one usually asks 'What is the intrinsic geometry of four-space?', the ether interpretation of gravity leads to the questions 'What is the intrinsic geometry of three-space?', 'What determines the ether velocity field?', and 'What is the velocity of light relative to the ether?'. Formally, these three questions are completely equivalent to the original one, but considering them separately nonetheless suggests ideas that are not usually considered in metric-space theories.

For example, considering the first question, it is readily apparent that three-space is very nearly Euclidean, and there is no observational evidence which indicates that it is not *exactly* Euclidean, so that the theory can be considerably simplified by considering only fields in which a Euclidean three-space exists. In such fields, the Lorentz invariant extension of Newtonian theory that has been given previously leads to a set of field equations involving the ether velocity and the velocity of light relative to the ether. These equations can be written in the form of equations (3.8) or equations (3.10), and they give an answer to the second question raised above. The third question above concerns the value of the function α, which can be determined only from the observed relativistic corrections to Newtonian theory. The corrections that have been verified in the Schwarzschild field suggest that α has the constant value c in this field, but if α is *always* equal to c the theory does not lead to gravitational radiation. The analogy between gravity and electromagnetism suggests that α does not *always* equal c but is approximately equal to c in large, nearly static fields, and this suggestion agrees well with the observational evidence. As a result, it is not unreasonable to assume that the velocity of light relative to the ether may have the constant value c in many physically important gravitational fields.

The magnitude of the ether velocity field is strongly dependent upon the choice of the time-like coordinate and can be made very small by a proper choice of the time variable. As a result, it is possible to introduce a new system of coordinates which are quite similar to the observer's coordinates in one Lorentz frame of the special theory, where the ether velocity vanishes identically. These are the coordinates $\bar{x}_\alpha$ given by equations (5.3). They have been defined so that they nearly reduce the metric tensor to the flat-space metric of the special theory, and it is then found that the coordinates $\bar{x}_\alpha$ in two different Lorentz frames are related to each other exactly by the usual Lorentz transformation of the special theory. This definition of the observer's coordinates is of importance in any attempt to unify physics, because all of physics, except gravity, is usually described in the coordinates of the special theory and cannot be directly compared with gravity until these coordinates have been defined in the gravitational field. Thus the emphasis put on the ether velocity field by interpreting gravity as an ether flow leads to an approach to the unification of physics which has not appeared previously in curved-space interpretations of gravity.

The interpretation of gravity given here leads to a space-time framework that is essentially the one of classical physics. In particular, there is an invariant time-like function t similar to Newton's universal time, and the three-space defined by one value of t is Euclidean. There is also an ether, as was often assumed in classical physics, but the ether differs from those of the classical theories in that its effect is more readily apparent in gravitation than in electromagnetism and its motion is determined by laws very different from those of classical hydrodynamics or elasticity, which were usually assumed to describe the ether in the classical theories. The resulting ether theory leads to Newtonian theory in the nonrelativistic limit and also leads exactly to the Schwarzschild field. As a result, when it is assumed that the equations of motion of particles and light rays are geodesics, the theory leads to all of the corrections to Newtonian theory which have been verified in the Schwarzschild field. It appears that the only observational evidence that is not described correctly is the existence of gravitational radiation, and it is not difficult to modify the theory so that it also predicts radiation. However, so little is known about the properties of gravitational radiation that the modification is not determined uniquely, and it is not discussed here.

Although the ether flow is formally equivalent to a curved metric four-space, the interpretation of the formalism is quite different from the one usually adopted in metric-space theories. This difference of interpretation can have a strong influence on the direction of future research, and could spell the difference between success and failure in a future physical theory.

References

Einstein, A. (1934). Relativity and the Ether. In: *Essays in Science*. Philosophical Library, New York.

Jánossy, L. (1966). The Lorentz Principle and the General Theory of Relativity, *Acta Physica*, **21**, 1, 17, 329.

Kirkwood, R. (1953). The Physical Basis of Gravitation. *Physical Review*, **92**(6), 1557.

Kirkwood, R. (1954). Gravitational Field Equations. *Physical Review*, **95**(4), 1051.

Kirkwood, R. (1970). Lorentz Invariance in a Gravitational Field. *Journal of Mathematical Physics*, **11**(10), 2983.

Kirkwood, R. (1972). A Metric-Space Formulation of Newtonian Fields. *International Journal of Theoretical Physics*, **6**, 133.